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ABSTRACTS

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About exceptional sets in Fenton's type theorem

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Let $\lambda = \{\lambda_n\} \subset \mathbb{R}_+$, $\beta = \{\beta_n\} \subset \mathbb{R}_+$, the sequences λ and $\alpha = \{\lambda_n + \beta_n\}$ be increasing, and $\tau : \mathbb{R}_+ \to \mathbb{R}_+$ be a nondecreasing function such that $\tau(x) \geq x$ $(x \geq 0)$. Denote by $S(\alpha, \beta, \tau)$ the class of convergent for all $x \in \mathbb{R}_+$ series of the form $F(z) = \sum_{n=0}^{+\infty} a_n e^{x\lambda_n + \tau(x)\beta_n}$, $a_n \geq 0$ $(n \geq 0)$. If $\sum_{n=0}^{+\infty} 1/(\lambda_{n+1} - \lambda_n) < +\infty$, then for every function $F \in S(\lambda, \beta, \tau)$ the relationship

$$F(x) = (1 + o(1))\mu(x, F) \tag{1}$$

holds as $x \to +\infty$ outside some set $E \subset [0, +\infty)$ of finite Lebesgue measure ([1]), where $\mu(x, F) = \sup\{a_n e^{x\lambda_n + \tau(x)\beta_n} : n \geq 0\}$. For $S(\lambda) = S(\lambda, 0, 0)$ (the class of entire Dirichlet series) it is ([2]) proved that for every sequence λ and every function $h : \mathbb{R}_+ \to \mathbb{R}_+$, $\frac{h(x)}{x} \to +\infty$ $(x \to +\infty)$, there exist a function $F \in S(\lambda)$, a set $E \subset [0, +\infty)$, a constant $d > 0$ such that $\int_E dh(x) = +\infty$ and

$$(\forall x \in E): \ F(x) > (1 + d)\mu(x, F). \tag{2}$$

Conjecture 1. The same is valid for the class $S(\alpha, \beta, \tau)$, i.e., for every sequences λ, β, functions τ, h, $\frac{h(x)}{x} \to +\infty$ $(x \to +\infty)$, there exist a function $F \in S(\lambda, \beta, \tau)$, a set E, a constant $d > 0$ such that $\int_E dh(x) = +\infty$ and $\forall x \in E$ inequality (2) holds.

Question. Let $h : \mathbb{R}_+ \to \mathbb{R}_+$, $\frac{h(x)}{x} \to +\infty$ $(x \to +\infty)$ and h be nondecreasing. What are necessary and sufficient conditions that the relationship (1) holds for $x \to +\infty$ ($x \notin E$, $\int_E dh(x) < +\infty$) for every function $F \in S(\alpha, \beta, \tau)$?

Let φ be positive continuously increasing to $+\infty$ on $[0; +\infty)$ function. Denote by $D^0_\varphi(\lambda) = \bigcap\limits_{0<\delta<1} D_{\delta\varphi}(\lambda)$, where

$$D_{\delta\varphi}(\lambda) = \{F \in D(\lambda) : (\exists K > 0)(\forall n \geq n_0) : |a_n| \leq e^{-\lambda_n \delta\varphi(K\lambda_n)}\}.$$

Theorem. *Let h, φ be positive continuously increasing to $+\infty$ on $[0; +\infty)$ functions, $h'(x)$ nonincreasing, and sequences λ such that*

$$\sum_{k=0}^{n} \frac{1}{\lambda_{k+1} - \lambda_k} = O(\varphi(\lambda_{n+1})) \quad (n \to +\infty),$$

$$(\exists q > 1): \ \sum_{n=1}^{+\infty} \frac{h'(q\varphi(\lambda_n))}{\lambda_n - \lambda_{n-1}} = +\infty.$$

There exists a function $F \in D^0_\varphi(\lambda)$, a set $E \subset [0, +\infty)$, a constant $d > 0$ such that $\int\limits_E dh(x) = +\infty$ and $\forall x \in E$ inequality (2) holds.

Conjecture 2. Let h, φ be positive continuously increasing to $+\infty$ on $[0; +\infty)$ functions, $h'(x)$ nonincreasing, and sequences λ such that $\sum\limits_{n=1}^{+\infty} h'(\varphi(\lambda_n))/(\lambda_n - \lambda_{n-1}) < +\infty$. Then for every function $F \in D_\varphi(\lambda)$ the relation (1) as $x \to +\infty$ ($x \notin E$, $\int_E dh(x) < +\infty$) holds.

1. Skaskiv O.B., Trusevych O.M. *On the exceptional sets in asymptotic equality between the sum and the maximal term of a positive series similar to the Taylor-Dirichlet series*, Mat. Metody Fiz.-Mekh. Polya, **45** (2002), no.1, 61–64. (in Ukrainian)
2. Salo T.M., Skaskiv O.B., Trakalo O.M. *On the best possible description of exceptional set in Wiman-Valiron theory for entire function*, Mat. Stud. **16** (2001), no.2, 131–140.